# Predictive and Prescriptive Analytics toward Optimizing Wildfire Suppression

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Acknowledgements: Ryne Reger and Erwin  Deng

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# An ever-growing impact of wildfires

**Los Angeles Times** January 7, 2025

**Pacific Palisades fire explodes to nearly 3,000 acres as thousands of residents flee, homes are lost**

**Los Angeles Times** January 9, 2025

**Death toll in Los Angeles wildfires rises to 10, officials report**

**The New York Times** January 11, 2025

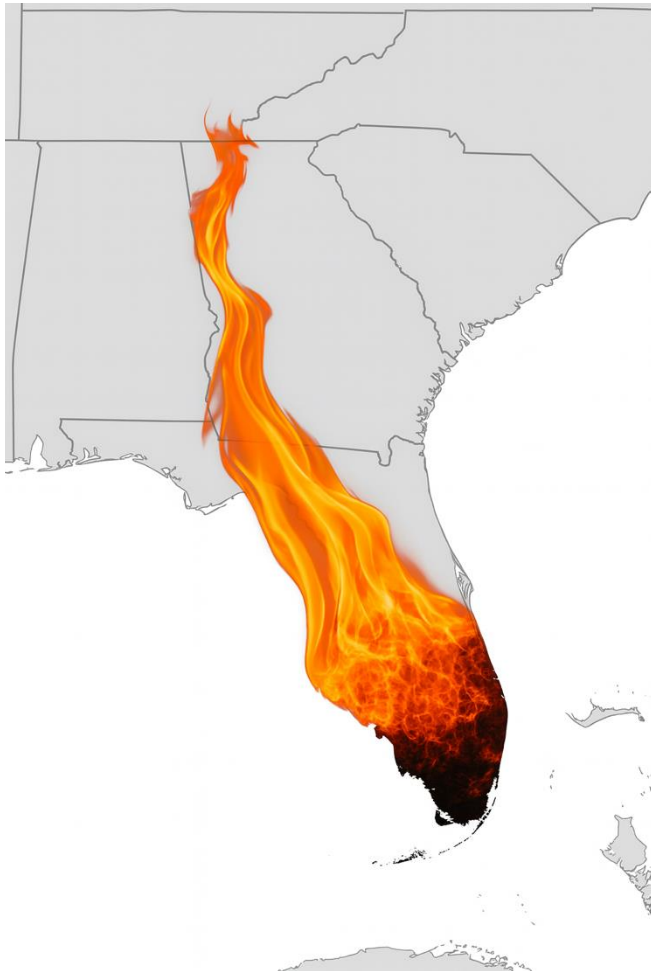
**Winds Intensify in L.A. as Wildfire Death Toll Rises to 16**

**"We are in a 'triage mode' where our primary focus must be on fires that threaten communities and infrastructure."**

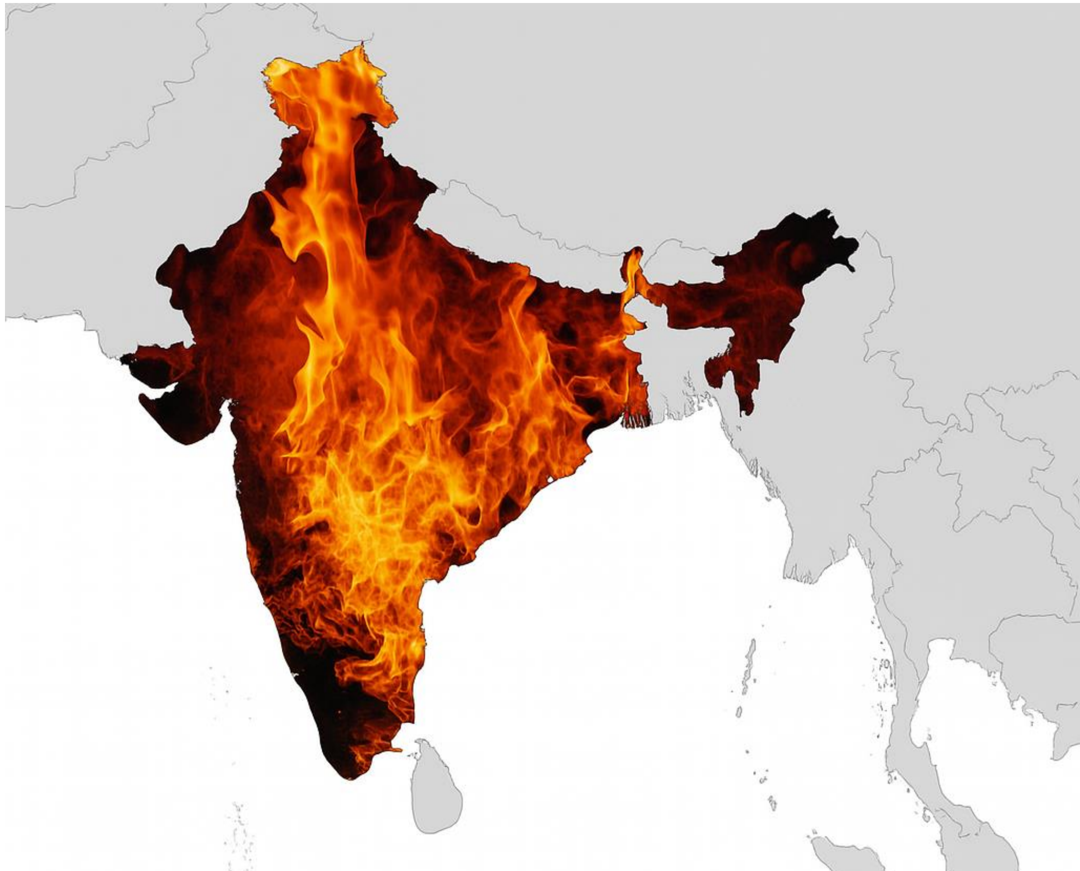
- Randy Moore, Chief
US Forest Service


Current Preparedness Level

In 2024, in the US, ~65,000 wildfires burned **~9 million acres.**



In 2024, wildfires globally burned approximately **~900 million acres** (3.88 million square kilometers).



# Simultaneous triage and routing

**Crew routing**


**Fire triage**

# Simultaneous triage and routing

**Crew routing**

- Vehicle routing-scheduling dynamics to attend fires in a spatial-temporal environment
- Routing considerations
  - Travel cost and time
  - Acute and accumulated fatigue
  - Policies and regulations
- **Endogenous demand**: Fires grow while unattended

**Fire triage**

- Triaging dynamics to prioritize high-risk fires, and delay suppression efforts elsewhere
- Triaging considerations
  - Damage and spread risk
  - Suppression difficulty
- **Heterogeneous switching costs** over a wide geography

# Contributions

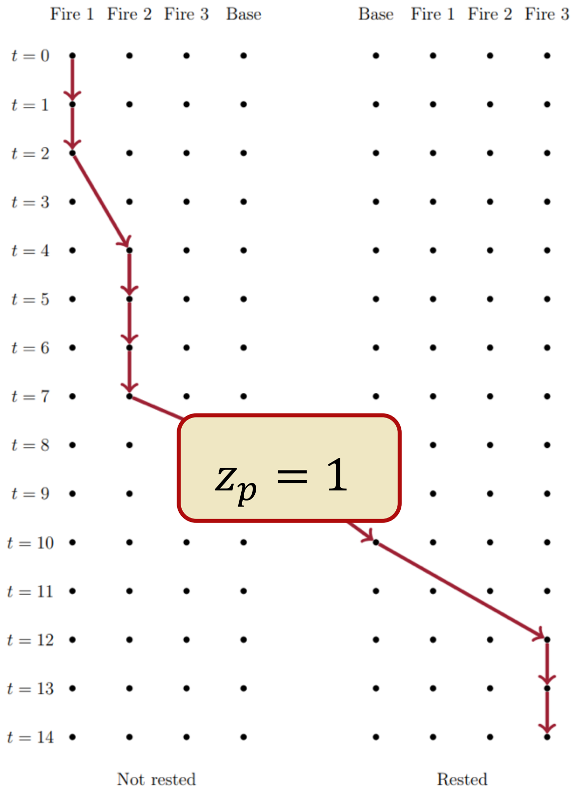
**Branch-and-price-and-cut algorithm for prescriptive wildfire analytics**

| | |
|---|---|
| Optimization modeling | Two-sided set-partition formulation to model fire spread dynamics and crew routes on time-expanded networks |
| Optimization algorithm | Two-sided branch-and-price-and-cut, with augmented GUB cutting planes, dual-aware branching, and primal heuristics |
| Computational results | Computational improvements in medium-scale problems, and provably high-quality solutions in large-scale problems |
| Data-driven case study | Data-driven impact: double machine learning to estimate the wildfire suppression dynamics, and benefits of optimization |


OPTIMIZING WILDFIRE SUPPRESSION

# Time-expanded network representation

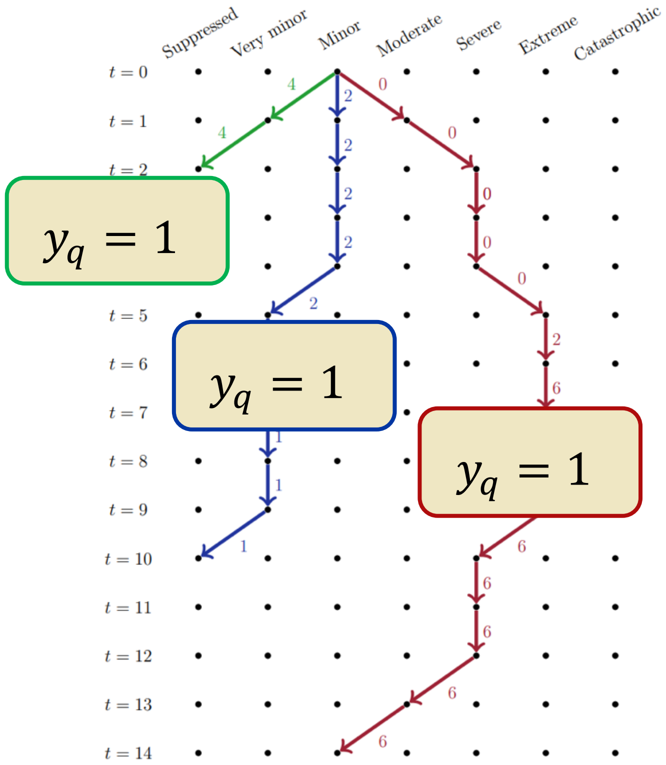
**Crews: time-space-rest network**




We model **full crew work plans** and **full fire evolution plans.**

We use **linking constraints** that tie these two networks together.

**Fires: time-state network**



# Two-sided set partitioning formulation

We avoid modeling individual arcs explicitly and instead model **entire plans & routes as decision variables**.

**Proposition: The formulations are equivalent, with the same relaxations**

$$
\begin{aligned}
\text{minimize} \quad & \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}_j} c_p z_p + \sum_{g \in \mathcal{G}} \sum_{q \in \mathcal{Q}_g} d_q y_q \\
\text{subject to} \quad & \sum_{q \in \mathcal{Q}_g} y_q = 1, \forall g \in \mathcal{G}, \\
& \sum_{p \in \mathcal{P}_j} z_p = 1, \forall j \in \mathcal{J}, \\
& \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}_j} A_{pgt} z_p - \sum_{q \in \mathcal{Q}_g} B_{qt} y_q \geq 0, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \\
& z_p \in \{0,1\}, \forall p \in \mathcal{P}_j, \ \forall j \in \mathcal{J}, \\
& y_q \in \{0,1\}, \forall q \in \mathcal{Q}_g, \ \forall g \in \mathcal{G}
\end{aligned}
$$

**Minimize travel cost and fatigue** (first objective term)

**Minimize fire damage** (second objective term)

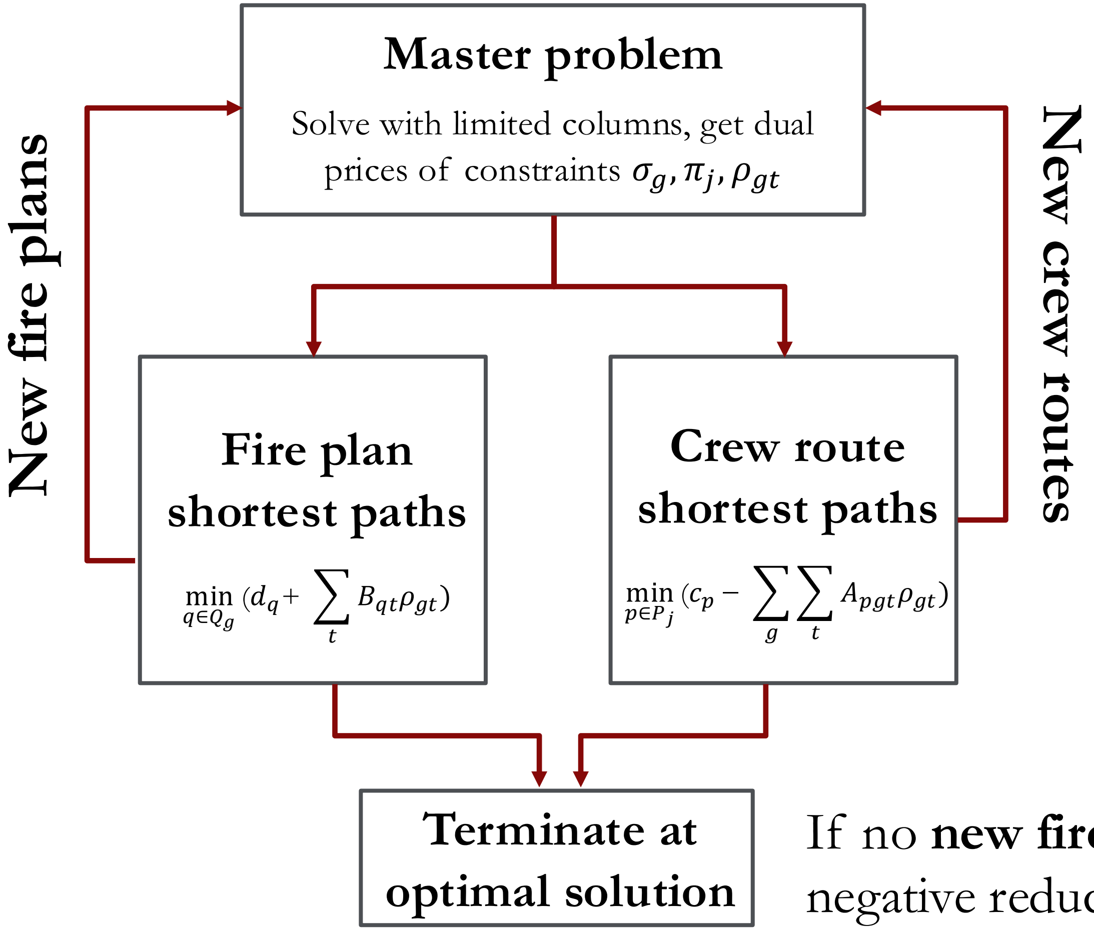
**One suppression plan per fire** (first constraint)

**One route per crew** (second constraint)

**Linking constraint: crew supply meets suppression plan demand at all times** (third constraint)

# Two-sided column generation

We solve the master problem using only a subset of all possible fire plans and crew routes and add more iteratively.




If no **new fire plan** or **crew route** has negative reduced cost, we're done.

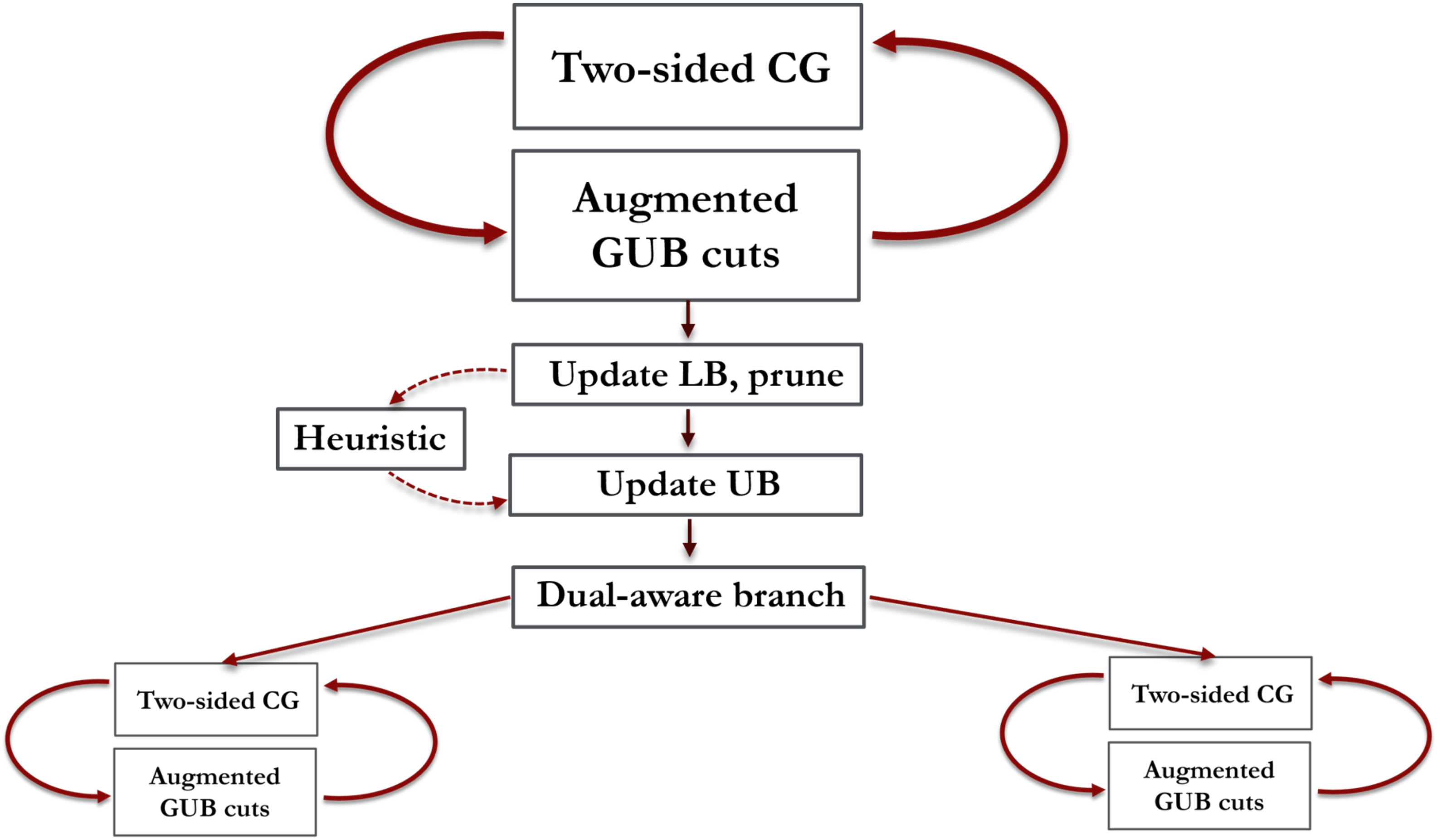
**The two-sided structure induces a decomposition by fire and by crew in the pricing problems**

**Key idea: $\boldsymbol{\rho_{gt}}$ incorporates local information into pricing problems**

# Putting it all together: Branch-and-price-and-cut

Key idea:

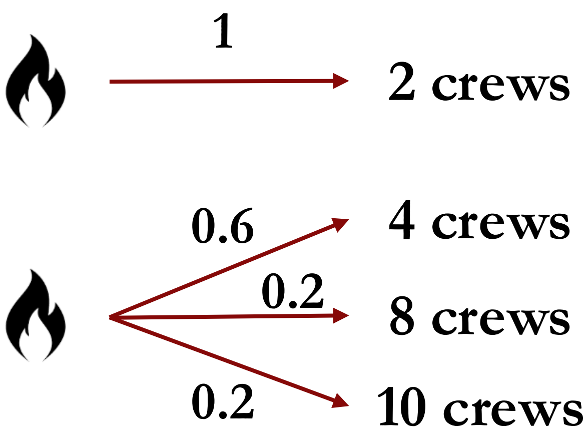
- We solve a smaller problem,
- **Generate** better fire plans and crew routes on demand,
- **Tighten** the problem with cuts,
- Use **heuristic** to find **feasible** integer solution,
- Intelligently **branch** when needed.

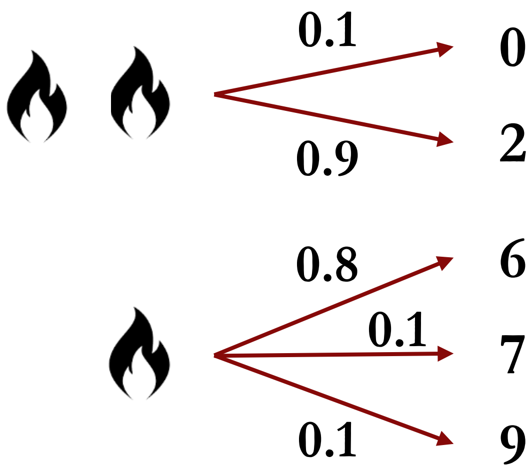



**Theorem: The algorithm converges finitely to an optimal solution**

# Cutting planes: 10-crew fire suppression plan example

Challenge: During master LP relaxation, the solver may pick **fractions of different fire suppression plans.**




$$\sum_{q\in\mathcal{Q}_1} \mathbb{1}[B_{qt} \geq 2] y_q + \sum_{q\in\mathcal{Q}_2} \mathbb{1}[B_{qt} \geq 10] y_q \leq 1$$

This is a **GUB cut**: *"You can't mix these plans because there's no way to round them into a valid integer solution."*

$$\frac{1}{2}\sum_{q\in\mathcal{Q}_1} \mathbb{1}[B_{qt} \geq 2] y_q + \frac{1}{2}\sum_{q\in\mathcal{Q}_2} \mathbb{1}[B_{qt} \geq 2] y_q + \frac{1}{2}\sum_{q\in\mathcal{Q}_3} \mathbb{1}[7 \leq B_{qt} \leq 8] y_q + \sum_{q\in\mathcal{Q}_3} \mathbb{1}[B_{qt} \geq 9] y_q \leq 1$$

# Cutting planes for LP: robust cuts

- Generalized upper-bound cover cuts (Wolsey)

$$\sum_{g\in\mathcal{G}_u}\sum_{q\in\mathcal{Q}_g}\mathbb{1}[B_{qt}\geq D_{ug}]y_q+\sum_{j\in\mathcal{J}_u}\sum_{p\in\mathcal{P}_j}\mathbb{1}\left[\sum_{g\in\mathcal{G}_u}A_{pgt}=0\right]z_p\leq|\mathcal{G}_u|+|\mathcal{J}_u|-1$$

- **Augmented GUB cover cuts**
  - Cut-generating linear program to generate augmented GUB cover cuts
  - Detects combinations of fire plans that are allowed in the LP and not eliminated by GUB cuts

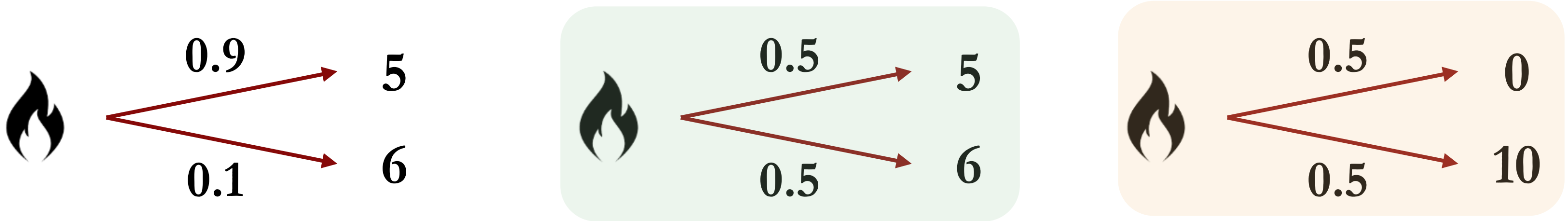
$$\sum_{g\in\mathcal{G}_u}\sum_{q\in\mathcal{Q}_g}\sum_{d=0}^{J}\mathbb{1}[B_{qt}=d]\delta_{ugd}y_q+\sum_{j\in\mathcal{J}_u}\sum_{p\in\mathcal{P}_j}\mathbb{1}\left[\sum_{g\in\mathcal{G}_u}A_{pgt}=0\right]z_p\leq|\mathcal{J}_u|+K_u$$

These cuts dramatically improve relaxation strength and reduce the number of branches needed.

# Branching: variable selection

Once the LP+cuts are as tight as possible, we still may need to **branch** to get an integer solution.

- Most fractional branching vs. maximum-variance branching




- **Dual-aware maximum variance variable selection**: leverage linking dual prices to identify "high-leverage" ambiguous variables (how valuable a crew is at that time and place).

$$\left(\sum_{q\in\mathcal{Q}_g} B_{qt}^2 y_q^*\right) - \left(\sum_{q\in\mathcal{Q}_g} B_{qt} y_q^*\right)^2 \longrightarrow \left(\sum_{q\in\mathcal{Q}_g} (B_{qt}\rho_{gt}^*)^2 y_q^*\right) - \left(\sum_{q\in\mathcal{Q}_g} B_{qt}\rho_{gt}^* y_q^*\right)^2$$

**Theorem: (Dual-aware) maximum-variance branching converges finitely to integer solution, whereas fractional branching may not.**

# Branch-and-price-and-cut algorithm

Benefits of augmented GUB cuts

Benefits of dual-aware branching

Benefits of primal heuristic

| Crews | Fires | Cuts | Heuristic | Branching | CPU | Opt. gap | UB | LB | Nodes | Columns |
|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 6 | False | False | MF | 1,200 | — | — | 1.00 | 4,757 | 600,982 |
| | | | | D-MV | 288 | 0.00% | 1.00 | 1.00 | 1,439 | 152,280 |
| | | | True | MF | 1,200 | 1.08% | 1.01 | 1.00 | 4,712 | 623,146 |
| | | | | D-MV | 313 | 0.00% | 1.00 | 1.00 | 1,437 | 170,650 |
| | | True | False | MF | 1,200 | — | — | 1.00 | 660 | 134,921 |
| | | | | D-MV | 229 | 0.00% | 1.00 | 1.00 | 83 | 24,118 |
| | | | True | MF | 1,200 | 0.65% | 1.01 | 1.00 | 642 | 137,101 |
| | | | | D-MV | 179 | 0.00% | 1.00 | 1.00 | 49 | 17,415 |
| 30 | 9 | False | False | MF | 1,200 | — | — | 1.00 | 1,832 | 410,675 |
| | | | | D-MV | 1,200 | — | — | 1.01 | 1,902 | 493,830 |
| | | | True | MF | 1,200 | 4.58% | 1.05 | 1.00 | 1,681 | 378,326 |
| | | | | D-MV | 1,200 | 2.84% | 1.04 | 1.02 | 1,682 | 433,931 |
| | | True | False | MF | 1,200 | — | — | 1.00 | 101 | 55,575 |
| | | | | D-MV | 1,200 | — | — | 1.02 | 135 | 66,109 |
| | | | True | MF | 1,200 | 3.57% | 1.04 | 1.00 | 52 | 58,285 |
| | | | | D-MV | 1,200 | 2.59% | 1.04 | 1.01 | 62 | 55,285 |

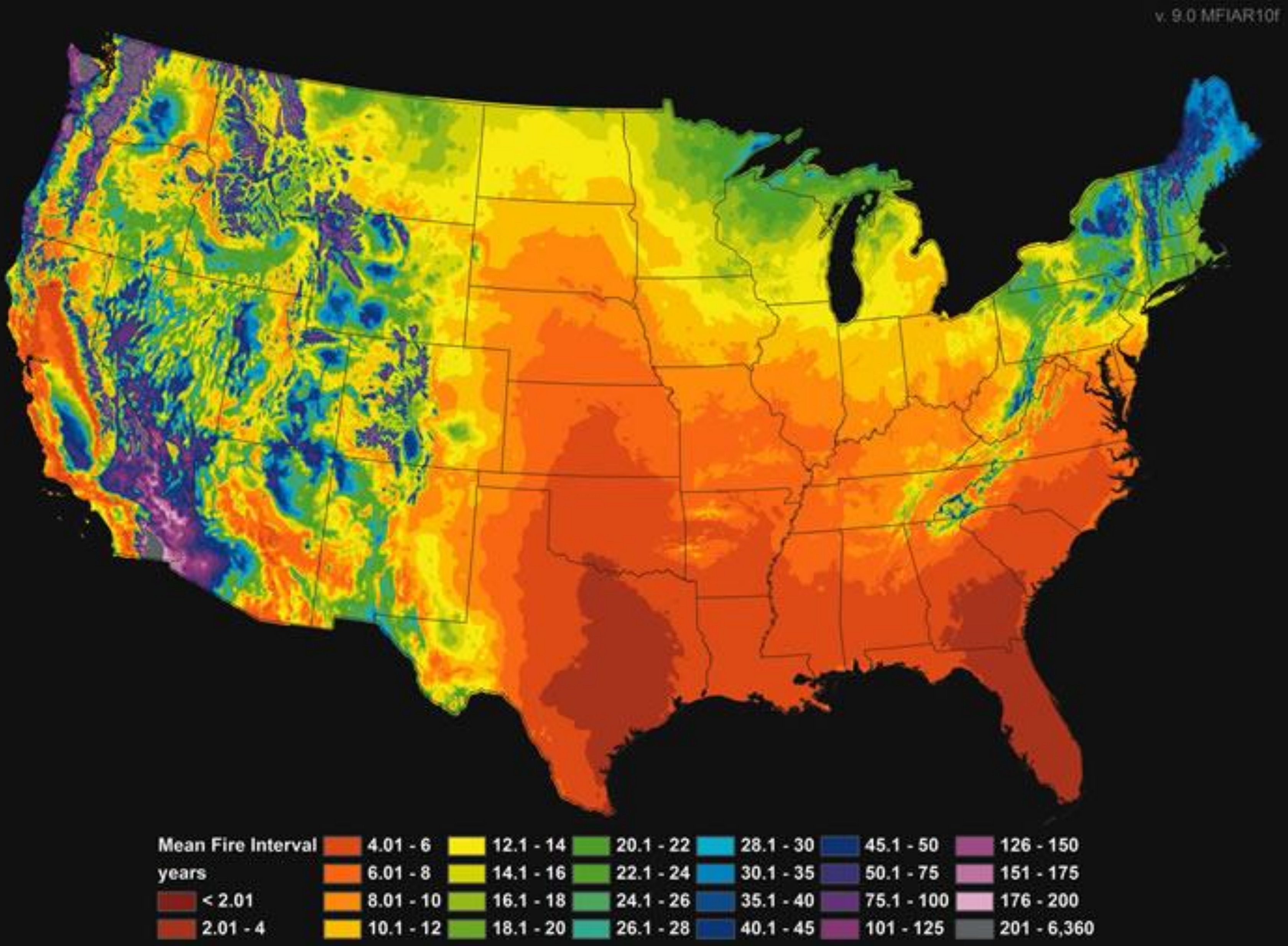
MF = most fractional branching, D-MV = dual-aware maximum-variance branching

# Overall performance assessment

- Poor scalability of off-the-shelf integer optimization methods
- Poor performance of simple 2CG+IP; in comparison, strong performance improvements from upper-bounding heuristic
- **Scalability of B&P&C: high-quality solutions in large instances**

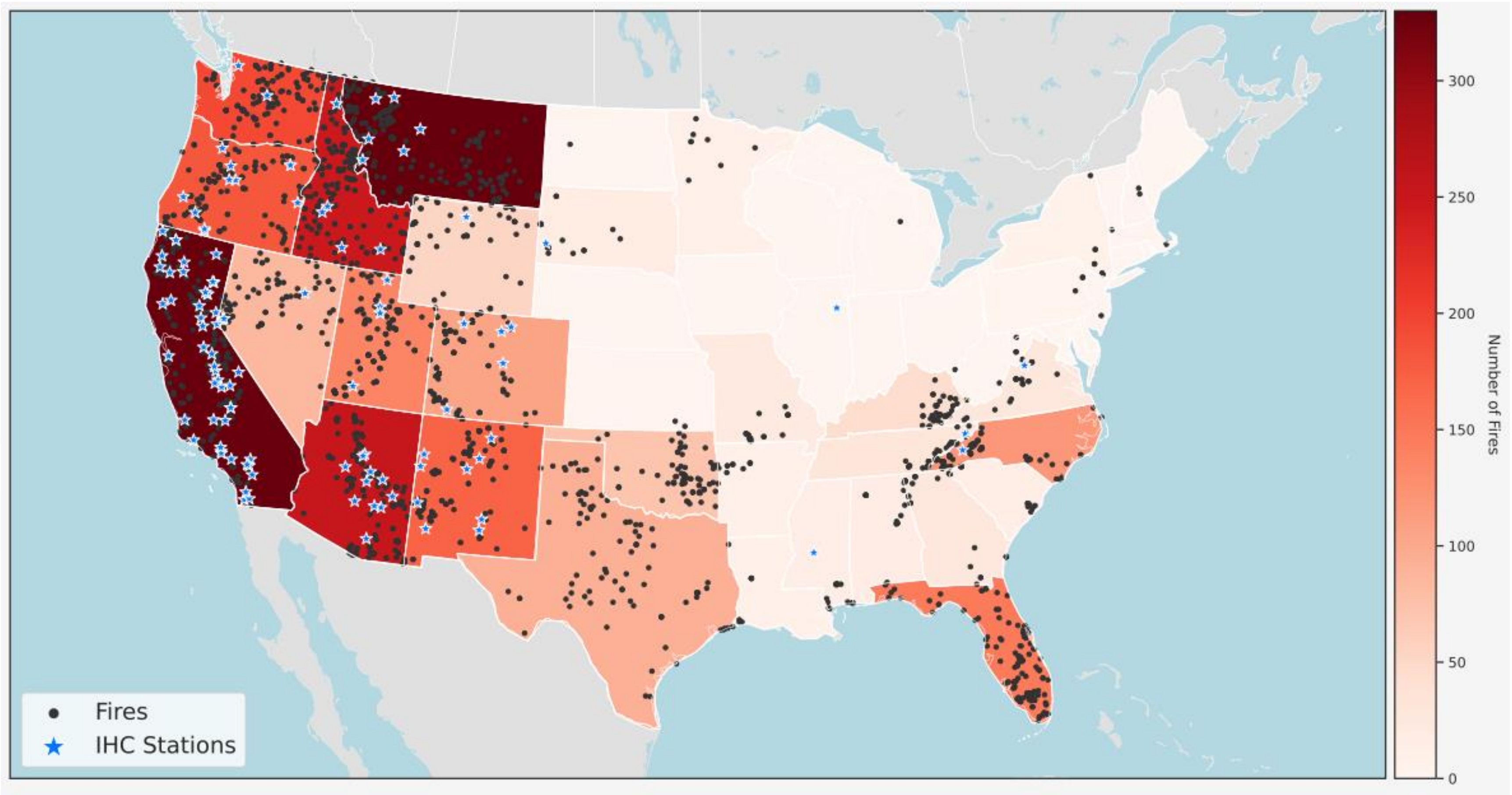
| | | Gurobi MIP solver | | | 2CG + IP | | | Root node + heuristic | | | Branch-and-price-and-cut | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C | F | UB | CPU | Opt. gap | UB | CPU | Opt. gap | UB | CPU | Opt. gap | UB | CPU | Opt. gap |
| 10 | 3 | 1.44 | 2 | 0.00% | 1.44 | 3 | 0.09% | 1.44 | 3 | 0.09% | 1.44 | 3 | 0.00% |
| 20 | 6 | 2.29 | 1,200 | 0.00% | 2.88 | 34 | 27.8% | 2.32 | 22 | 2.94% | 2.29 | 179 | 0.00% |
| 30 | 9 | 4.86 | 1,200 | 6.22% | 8.91 | 509 | 99.1% | 4.73 | 108 | 5.77% | 4.70 | 1,200 | 2.59% |
| 40 | 12 | 6.52 | 1,200 | 35.7% | 8.74 | 1,200 | 84.1% | 5.00 | 206 | 5.24% | 4.95 | 1,200 | 2.84% |
| 50 | 15 | 7.17 | 1,200 | 23.9% | 513 | 1,200 | 8.83e3% | 6.15 | 137 | 7.05% | 5.91 | 1,200 | 1.74% |
| 60 | 18 | 12.17 | 1,200 | 81.2% | 515 | 1,200 | 7.62e3% | 7.14 | 133 | 7.12% | 6.97 | 1,200 | 3.76% |
| 70 | 21 | 1.40e5 | 1,200 | 2.00e6% | 9.71 | 1,200 | 39.1% | 7.45 | 166 | 6.76% | 7.40 | 1,200 | 5.53% |

# Data-driven case study

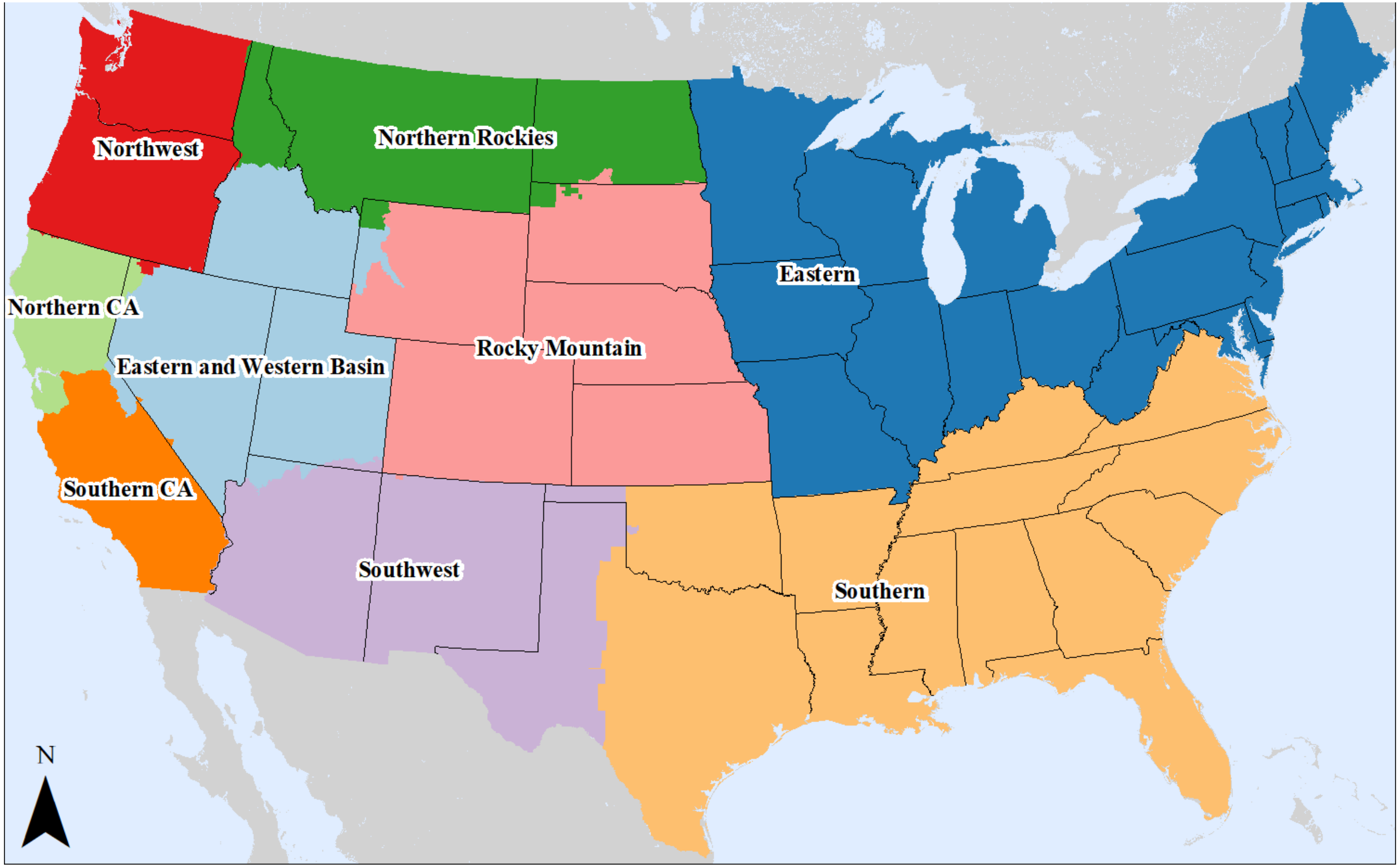


# Case study setting

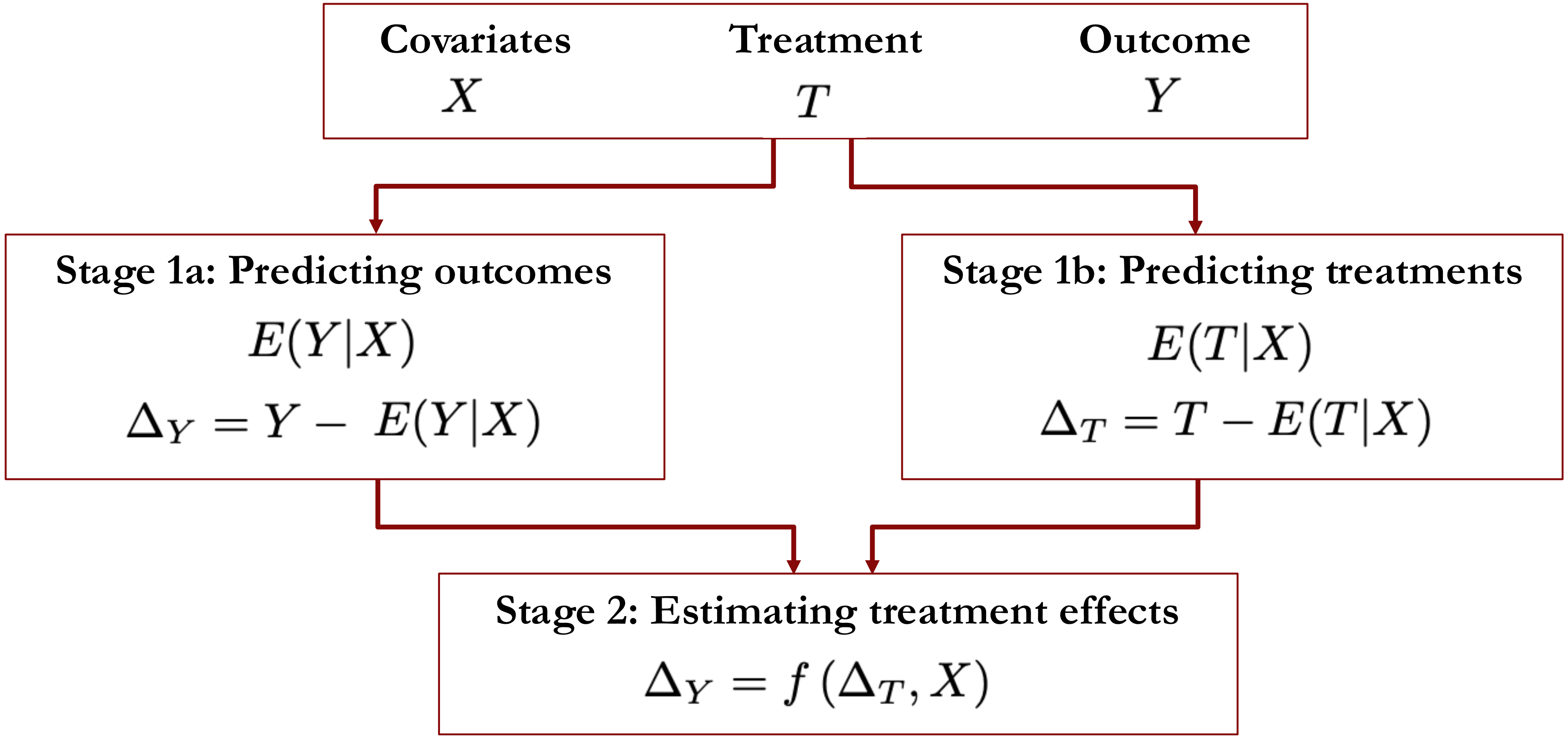
- Data-driven setup based on historical wildfire activity in the United States.
- Data on historical fires, environmental factors, and crew duties
- ~110 hot-shot crews (~ 20 firefighters/crew)
- 10–50 simultaneous fires at peak season.



# US divided into Geographic Area Coordination Centers (GACCs)



# Double machine learning framework




How much does sending an extra crew reduce fire growth, controlling for confounding?

**Athey and Wager. Estimating treatment effects with causal forests: An application, 2019.**
**Chernozhukov et al., Automatic debiased machine learning of causal and structural effects, 2022.**

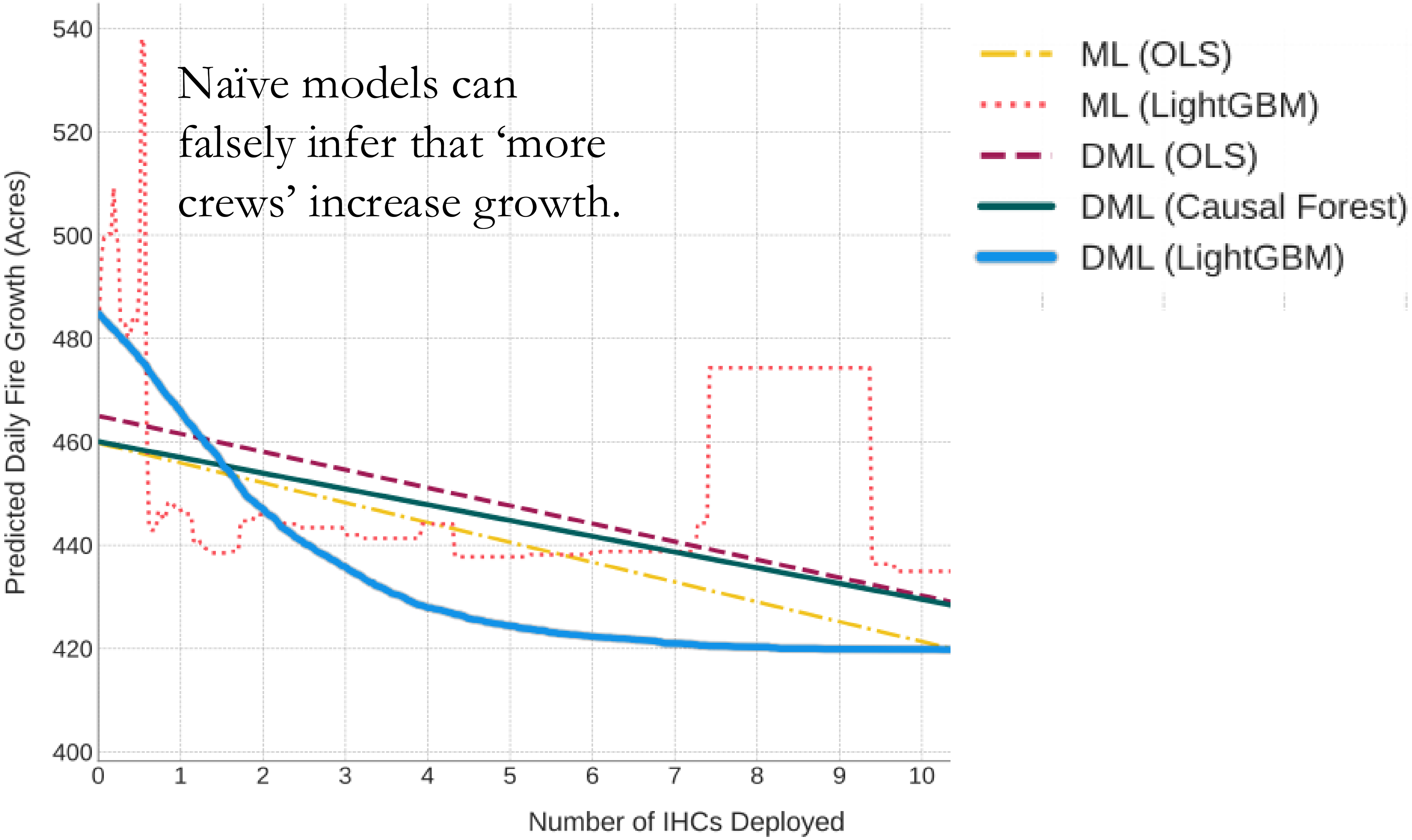
# Benefits of double machine learning

1. Stronger out-of-sample performance by isolating the impact of historical suppression efforts on wildfire growth
2. Treatment effect estimates for wildfire suppression optimization

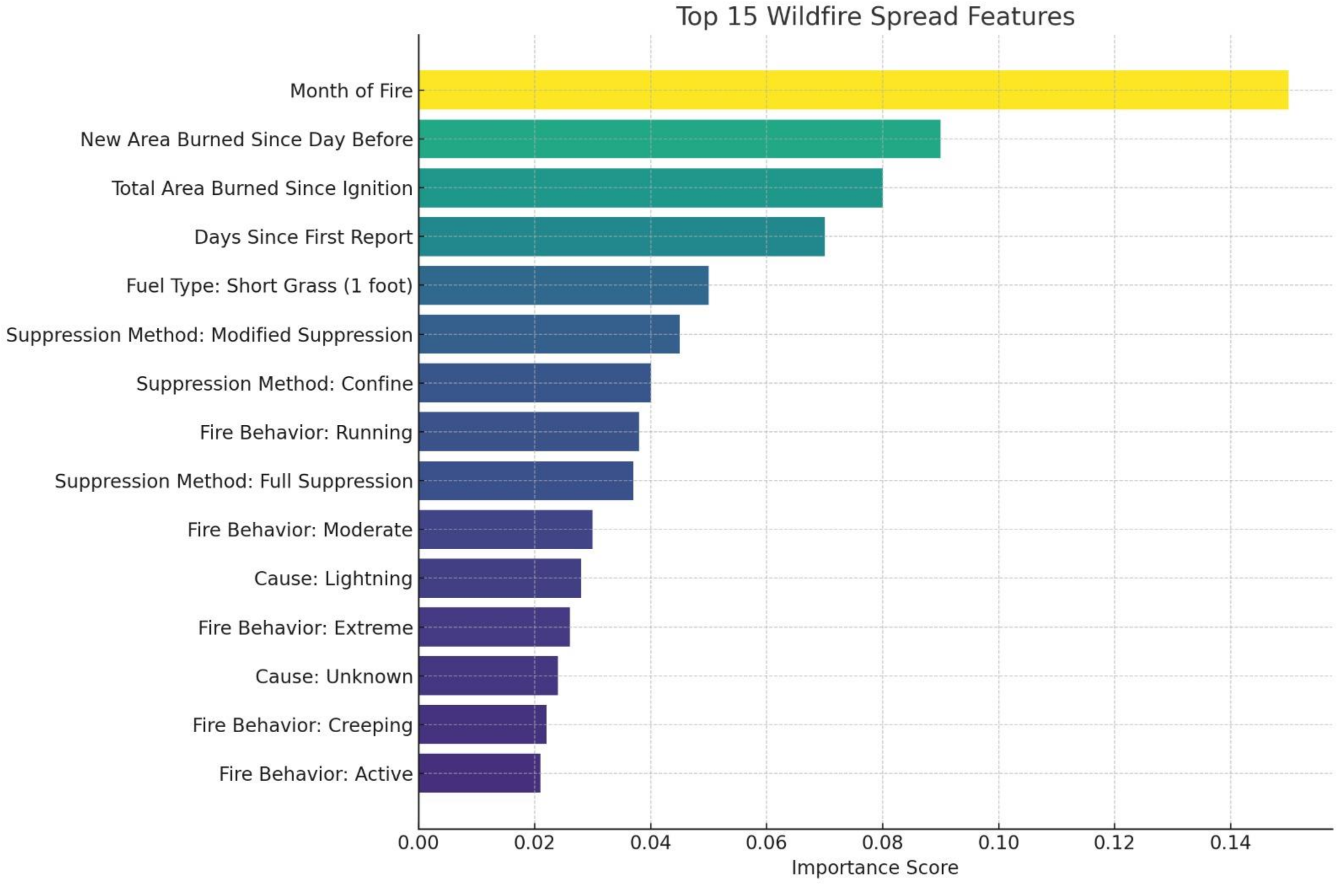
| Model | RMSE | MAE | $R^2$ | 90% shortfall |
|---|---|---|---|---|
| *Single-stage machine learning* | | | | |
| Ordinary Least Square (OLS) | 599.7 | 424.2 | 0.127 | 1,468.5 |
| **LightGBM** | **584.4** | **405.3** | **0.171** | **1,427.8** |
| *Double machine learning* | | | | |
| LightGBM + OLS | 590.3 | **413.9** | 0.154 | 1,400.1 |
| LightGBM + Causal Forest | 596.3 | 419.2 | 0.137 | 1,447.6 |
| **LightGBM + LightGBM** | **589.8** | 415.4 | **0.156** | **1,389.9** |

DML nomenclature: First-stage model + second-stage model

# Why we want double machine learning



# Feature importance

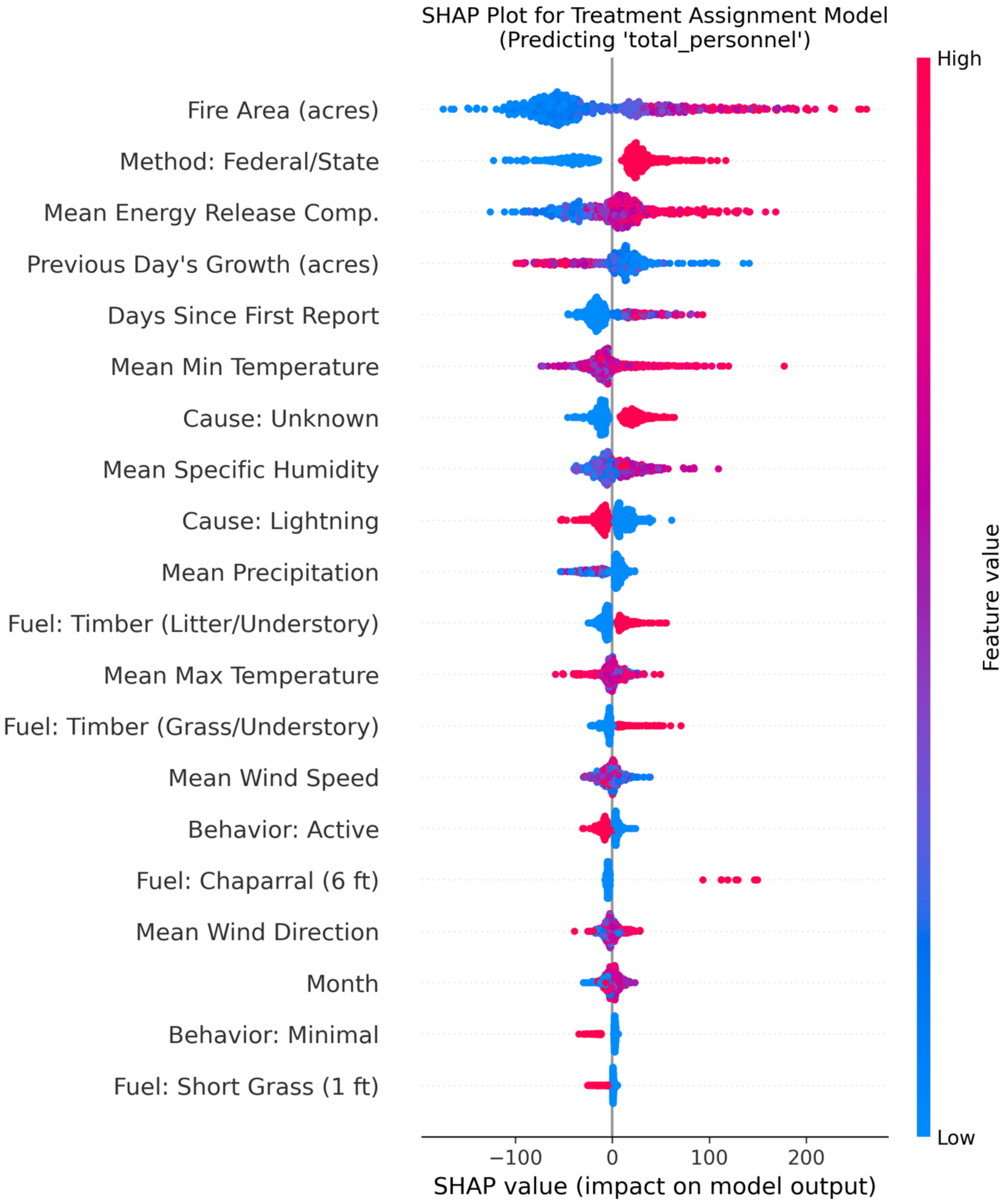



**Time of year**

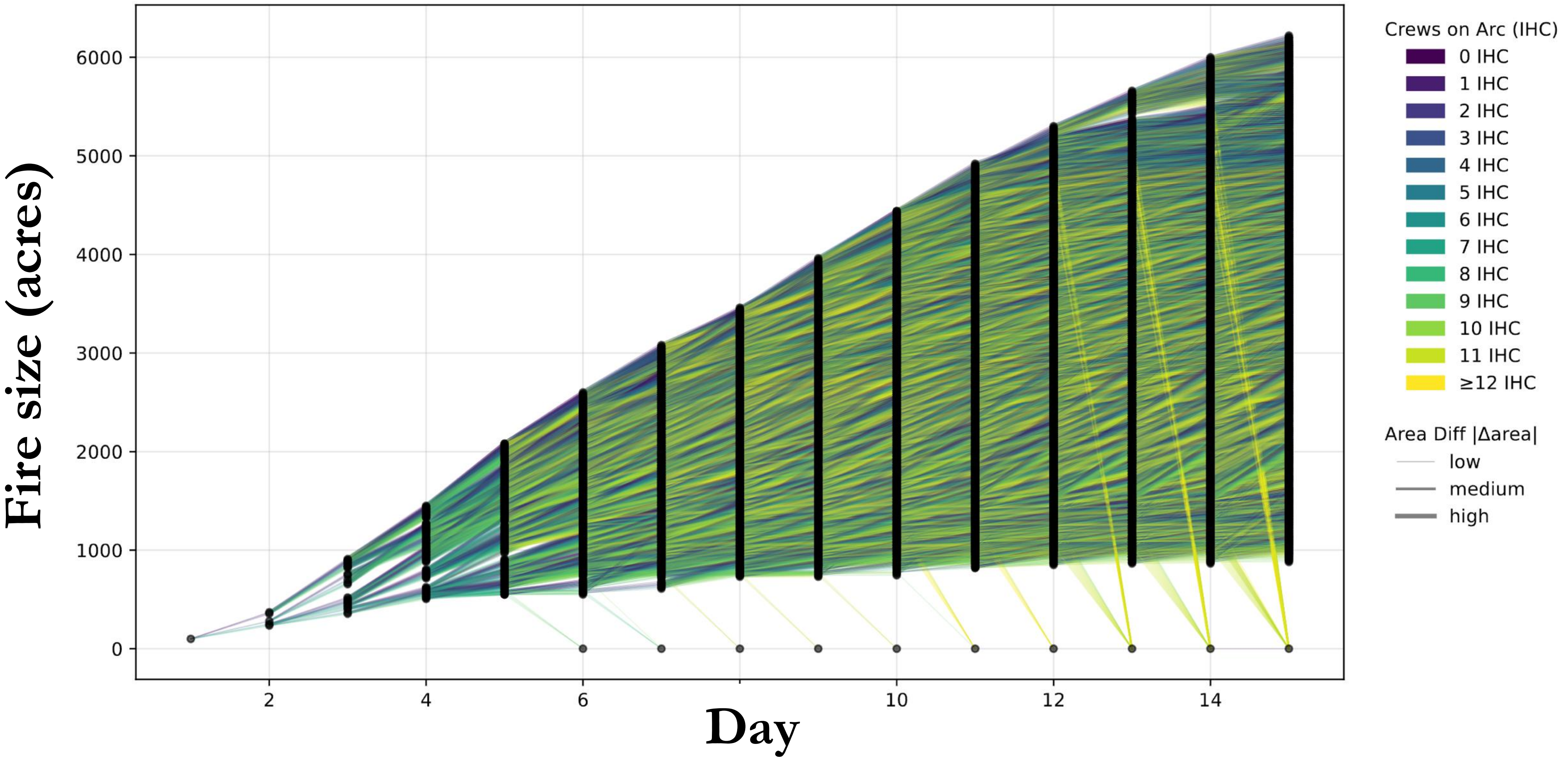
**Fire history**

**Environmental factors**

# What drives higher personnel allocation?

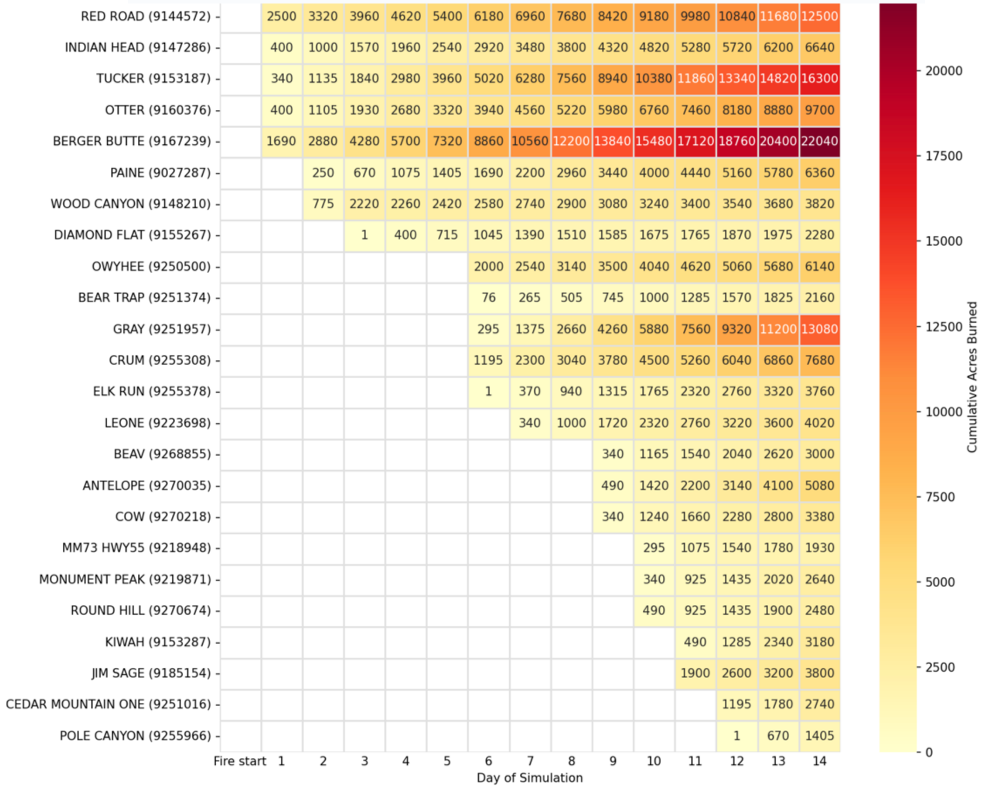


# Data-driven time-state fire networks

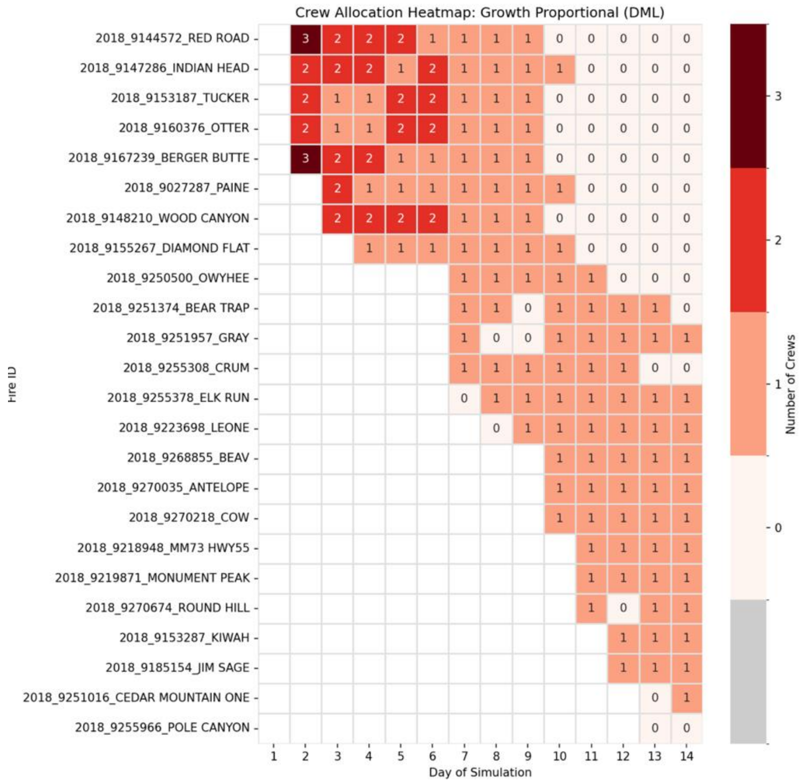


# Results visualization

## Wildfire progression over time

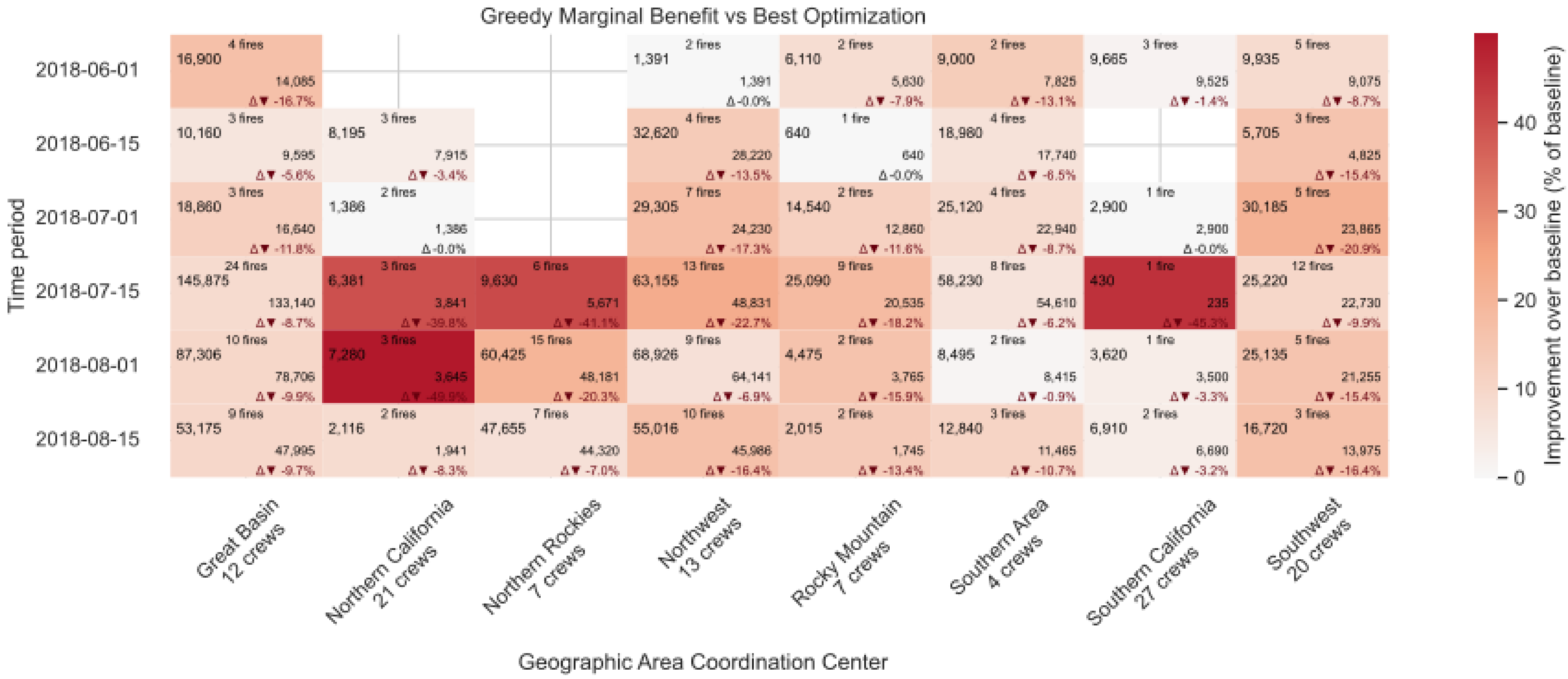



## Crew assignments over time




**Benefits of optimization: fine-tuning crew allocations based on wildfire impact and spatial-temporal propagation dynamics.**

# Optimization benefits across time and GACCs




| Time period | Great Basin 12 crews | Northern California 21 crews | Northern Rockies 7 crews | Northwest 13 crews | Rocky Mountain 7 crews | Southern Area 4 crews | Southern California 27 crews | Southwest 20 crews |
|---|---|---|---|---|---|---|---|---|
| 2018-06-01 | 4 fires 16,900 14,085 Δ▼ -16.7% | | | 2 fires 1,391 1,391 Δ -0.0% | 2 fires 6,110 5,630 Δ▼ -7.9% | 2 fires 9,000 7,825 Δ▼ -13.1% | 3 fires 9,665 9,525 Δ▼ -1.4% | 5 fires 9,935 9,075 Δ▼ -8.7% |
| 2018-06-15 | 3 fires 10,160 9,595 Δ▼ -5.6% | 3 fires 8,195 7,915 Δ▼ -3.4% | | 4 fires 32,620 28,220 Δ▼ -13.5% | 1 fire 640 640 Δ -0.0% | 4 fires 18,980 17,740 Δ▼ -6.5% | | 3 fires 5,705 4,825 Δ▼ -15.4% |
| 2018-07-01 | 3 fires 18,860 16,640 Δ▼ -11.8% | 2 fires 1,386 1,386 Δ -0.0% | | 7 fires 29,305 24,230 Δ▼ -17.3% | 2 fires 14,540 12,860 Δ▼ -11.6% | 4 fires 25,120 22,940 Δ▼ -8.7% | 1 fire 2,900 2,900 Δ -0.0% | 5 fires 30,185 23,865 Δ▼ -20.9% |
| 2018-07-15 | 24 fires 145,875 133,140 Δ▼ -8.7% | 3 fires 6,381 3,841 Δ▼ -39.8% | 6 fires 9,630 5,671 Δ▼ -41.1% | 13 fires 63,155 48,831 Δ▼ -22.7% | 9 fires 25,090 20,535 Δ▼ -18.2% | 8 fires 58,230 54,610 Δ▼ -6.2% | 1 fire 430 235 Δ▼ -45.3% | 12 fires 25,220 22,730 Δ▼ -9.9% |
| 2018-08-01 | 10 fires 87,306 78,706 Δ▼ -9.9% | 3 fires 7,280 3,645 Δ▼ -49.9% | 15 fires 60,425 48,181 Δ▼ -20.3% | 9 fires 68,926 64,141 Δ▼ -6.9% | 2 fires 4,475 3,765 Δ▼ -15.9% | 2 fires 8,495 8,415 Δ▼ -0.9% | 1 fire 3,620 3,500 Δ▼ -3.3% | 5 fires 25,135 21,255 Δ▼ -15.4% |
| 2018-08-15 | 9 fires 53,175 47,995 Δ▼ -9.7% | 2 fires 2,116 1,941 Δ▼ -8.3% | 7 fires 47,655 44,320 Δ▼ -7.0% | 10 fires 55,016 45,986 Δ▼ -16.4% | 2 fires 2,015 1,745 Δ▼ -13.4% | 3 fires 12,840 11,465 Δ▼ -10.7% | 2 fires 6,910 6,690 Δ▼ -3.2% | 3 fires 16,720 13,975 Δ▼ -16.4% |

# Benefits of collaboration between GACCs

| Method | Total acres burned (Jun-Aug) | Acres saved vs. no crew | % acres saved vs. no crew |
|---|---|---|---|
| No crew | 1,185,981 | ___ | ___ |
| Baseline: Crews allocated based on expected area growth | 1,067,130 | 118,851 | 10.0% |
| Baseline: Crews allocated based on their proximity to fires | 1,048,738 | 137,243 | 11.6% |
| Baseline: Crews allocated based on their maximum impact | 1,049,777 | 136,204 | 11.5% |
| Optimization no coordination across GACCs | 918,665 | 267,316 | 22.5% |
| **Optimization full coordination across GACCs** | **681,511** | **504,470** | **42.5%** |

**"180,000 football fields saved" thanks to coordination**

# Conclusion

**Branch-and-price-and-cut algorithm for prescriptive wildfire analytics**

**Optimization modeling**

**Optimization algorithm**

**Computational results**

**Data-driven case study**

- We developed a tractable, scalable algorithm that jointly optimizes wildfire triage and crew routing.
- We combine prescriptive optimization with causal ML, and show strong improvements over existing methods, both in theory and practice.
- This can support coordination and policy planning at the inter-agency level.

# Thank you!

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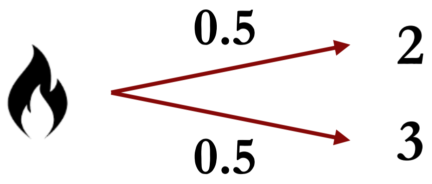
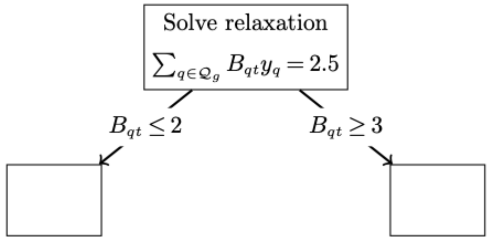
# Branching: preliminaries

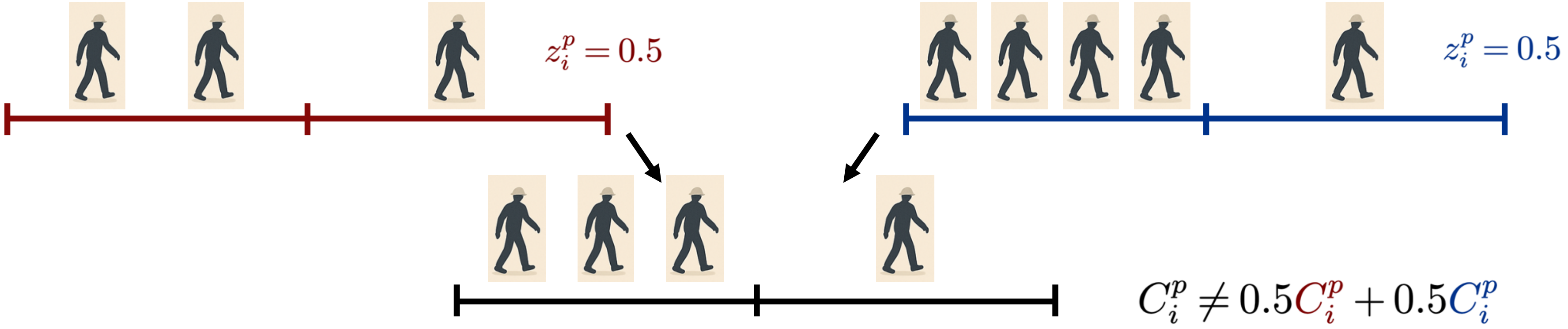
- Branching on “natural” variables: fire demand and crew assignments




$$\left(\sum_{q\in\mathcal{Q}_g} \mathbb{1}[B_{qt} > d]\, y_q = 0\right) \vee \left(\sum_{q\in\mathcal{Q}_g} \mathbb{1}[B_{qt} > d]\, y_q = 1\right)$$

$$\left(\sum_{p\in\mathcal{P}_j} \mathbb{1}[A_{pgt} = 1]\, z_p = 0\right) \vee \left(\sum_{p\in\mathcal{P}_j} \mathbb{1}[A_{pgt} = 1]\, z_p = 1\right)$$



# Natural formulation

$$
\begin{aligned}
\text{minimize} \quad & \sum_{j \in \mathcal{J}} \sum_{a \in \mathcal{A}_j} c_a z_a + \sum_{g \in \mathcal{G}} \sum_{a \in \mathcal{B}_g} d_a y_a \\
\text{subject to} \quad & \sum_{a \in \delta^+(n)} z_a - \sum_{a \in \delta^-(n)} z_a = 0, \ \forall j \in \mathcal{J}, \forall n \in \mathcal{N}_j \text{ s.t. } 0 < \tau(n) < T, \\
& \sum_{a \in \delta^-((l_j^0, 0, 0))} z_a = 1, \ \forall j \in \mathcal{J}, \\
& \sum_{a \in \delta_g^+(n)} y_a - \sum_{a \in \delta_g^-(n)} y_a = 0, \ \forall g \in \mathcal{G}, \forall n \in \mathcal{N}_g \text{ s.t. } 0 < \tau(n) < T, \\
& \sum_{a \in \delta^-((s_g^0, 0))} y_a = 1, \ \forall g \in \mathcal{G}, \\
& \sum_{j \in \mathcal{J}} \sum_{a \in \mathcal{A}_j^{gt}} z_a - \sum_{a \in \mathcal{B}_g^t} r_a y_a \geq 0, \ \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \\
& z_a \in \{0, 1\}, \ \forall j \in \mathcal{J}, \forall a \in \mathcal{A}_j, \\
& y_a \in \{0, 1\}, \ \forall g \in \mathcal{G}, \forall a \in \mathcal{B}_g.
\end{aligned}
$$

**Minimize total travel cost and fatigue**

**Minimize total fire damage**

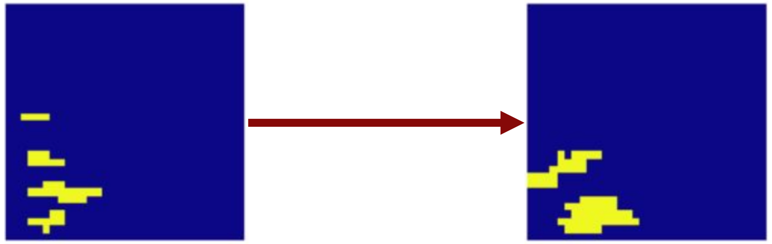
**Crew flow balance**

**Fire flow balance**

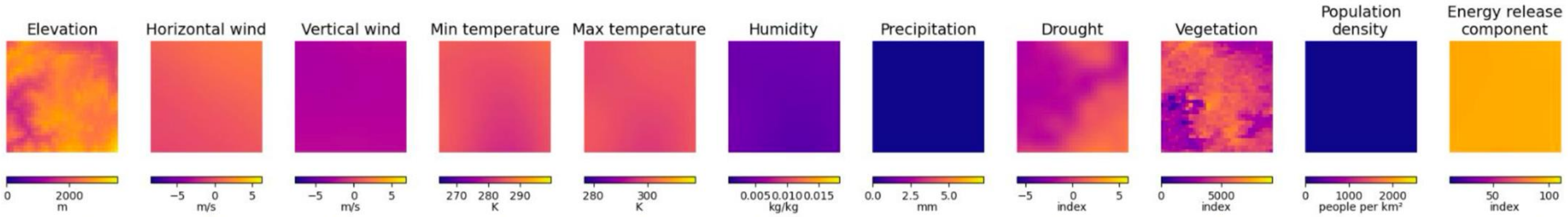
**Linking constraint: crew supply meets fire demand at all times**

# Wildfire data

- Visibility into historical wildfires and suppression efforts
  - Satellite data on daily wildfire mask [e.g., Google Earth Engine]



  - Covariate information: topologic and environmental data




  - New data on historical crew and machine assignments to wildfires

# Toward column generation

**Master problem primal**

$$\begin{aligned}
\text{minimize} \quad & \sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_j} c_p z_p + \sum_{g\in\mathcal{G}}\sum_{q\in\mathcal{Q}_g} d_q y_q \\
\text{subject to} \quad & \sum_{q\in\mathcal{Q}'_g} y_q = 1,\ \forall g\in\mathcal{G}, \\
& \sum_{p\in\mathcal{P}'_j} z_p = 1,\ \forall j\in\mathcal{J}, \\
& \sum_{j\in\mathcal{J}}\sum_{p\in\mathcal{P}_j} A_{pgt} z_p - \sum_{q\in\mathcal{Q}_g} B_{qt} y_q \geq 0,\ \forall g\in\mathcal{G}, \forall t\in\mathcal{T}, \\
& z_p \geq 0, \forall p\in\mathcal{P}'_j,\ \forall j\in\mathcal{J}, \\
& y_q \geq 0, \forall q\in\mathcal{Q}'_g,\ \forall g\in\mathcal{G}
\end{aligned}$$

**Master problem dual**

$$\begin{aligned}
\text{maximize} \quad & \sum_{g\in\mathcal{G}} \sigma_g + \sum_{j\in\mathcal{J}} \pi_j \\
\text{subject to} \quad & \sigma_g - \sum_{t\in\mathcal{T}} B_{qt}\rho_{gt} \leq d_q,\ \forall g\in\mathcal{G}, \forall q\in\mathcal{Q}'_g, \\
& \pi_j + \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} A_{pgt}\rho_{gt} \leq c_p,\ \forall j\in\mathcal{J}, \forall p\in\mathcal{P}'_j, \\
& \rho_{gt} \geq 0,\ \forall g\in\mathcal{G}, \forall t\in\mathcal{T}
\end{aligned}$$

**The dual constraints induce a decomposition by fire and by crew**

# Capacitated fire demand heuristic

$$\overline{D}_{gt} = \left\lceil \sum_{q \in \mathcal{Q}_g} B_{qt} y_q^* \right\rceil$$

$$\begin{aligned}
\text{minimize} \quad & \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}_j} c_p z_p + \sum_{g \in \mathcal{G}} \sum_{q \in \mathcal{Q}_g} d_q y_q \\
\text{subject to} \quad & \sum_{q \in \mathcal{Q}'_g} y_q = 1, \forall g \in \mathcal{G}, \quad \sum_{q \in \mathcal{Q}'_g} \mathbb{1}[B_{qt} \leq \overline{D}_{gt}] y_q^* = 1, \ \forall g \in \mathcal{G}, \forall t \in \mathcal{T} \\
& \sum_{p \in \mathcal{P}'_j} z_p = 1, \forall j \in \mathcal{J}, \\
& \sum_{j \in \mathcal{J}} \sum_{p \in \mathcal{P}_j} A_{pgt} z_p - \sum_{q \in \mathcal{Q}_g} B_{qt} y_q \geq 0, \forall g \in \mathcal{G}, \forall t \in \mathcal{T}, \\
& z_p \geq 0, \forall p \in \mathcal{P}'_j, \ \forall j \in \mathcal{J}, \\
& y_q \geq 0, \forall q \in \mathcal{Q}'_g, \ \forall g \in \mathcal{G}
\end{aligned}$$

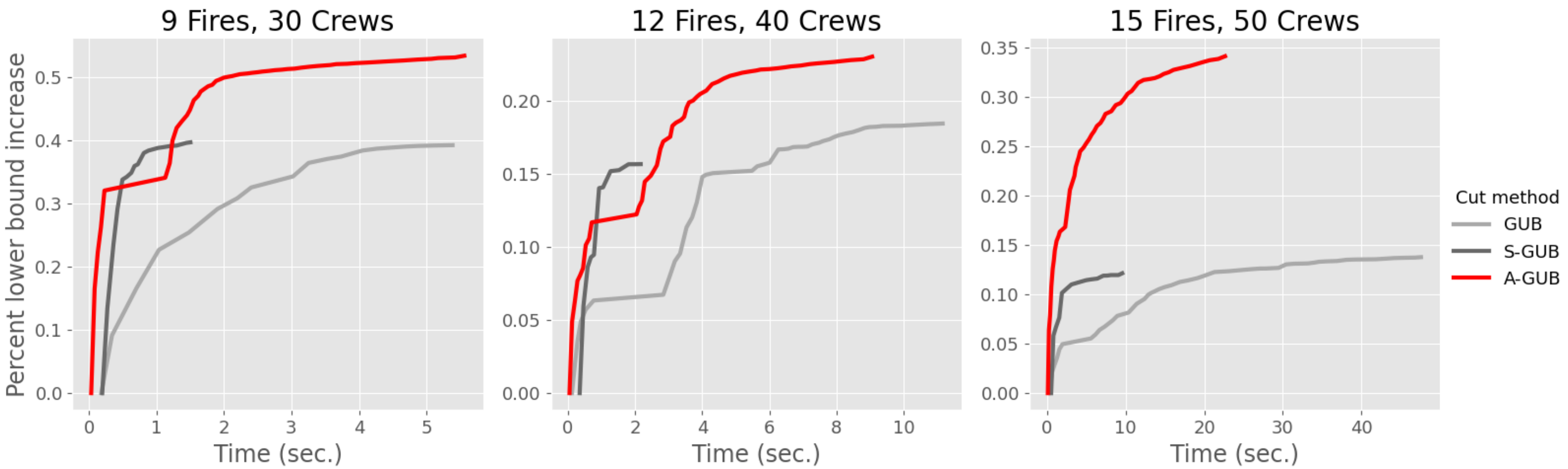
# Two-sided column generation

- Convergence with a small subset of fire plans and crew routes
- Tractability improvements: 75–90% speedups vs. direct implementation of the (polynomial) natural formulation
- Scalability to large-scale problems, albeit with optimality gap

| Instance size | | | | | CPU (seconds) | | | | | 2CG information | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Crews | Fires | F states | F arcs | C arcs | Direct | 2CG | RMP | FSP | CSP | Iter. | F cols. | C cols. |
| 10 | 3 | 416 | 15,655 | 4,664 | 0.43 | 0.16 | 0.01 | 0.06 | 0.07 | 46 | 102 | 220 |
| 20 | 6 | 782 | 63,259 | 27,868 | 1.78 | 0.41 | 0.13 | 0.18 | 0.08 | 42 | 170 | 537 |
| 30 | 9 | 1,228 | 136,639 | 84,732 | 6.99 | 0.97 | 0.14 | 0.59 | 0.23 | 56 | 291 | 812 |
| 40 | 12 | 1,644 | 200,984 | 190,376 | 14.39 | 1.39 | 0.25 | 0.57 | 0.56 | 49 | 323 | 1,320 |
| 50 | 15 | 2,010 | 269,114 | 359,920 | 22.79 | 2.47 | 0.39 | 0.88 | 1.15 | 45 | 415 | 1,873 |
| 60 | 18 | 2,276 | 315,910 | 608,484 | 33.91 | 4.34 | 0.88 | 1.36 | 2.08 | 61 | 622 | 3,344 |
| 70 | 21 | 2,692 | 377,978 | 951,188 | 62.10 | 4.75 | 0.85 | 1.10 | 2.69 | 49 | 566 | 3,067 |

# Benefits of augmented GUB cuts

- Augmented GUB cuts improve upon GUB cuts
  - Tighter relaxation: >2x increase in lower bound improvements
  - Faster convergence: >2x reductions in computational times (enabled by a cut-generating linear program)

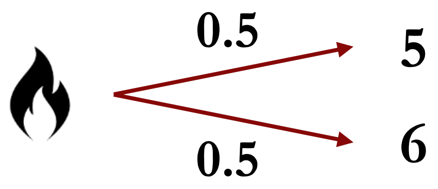


# Branching rules

- Natural branching rules: fire demands and crew assignments

$$\left(\sum_{q\in\mathcal{Q}_g} \mathbb{1}[B_{qt} > d]\, y_q = 0\right) \vee \left(\sum_{q\in\mathcal{Q}_g} \mathbb{1}[B_{qt} > d]\, y_q = 1\right)$$

$$\left(\sum_{p\in\mathcal{P}_j} \mathbb{1}[A_{pgt} = 1]\, z_p = 0\right) \vee \left(\sum_{p\in\mathcal{P}_j} \mathbb{1}[A_{pgt} = 1]\, z_p = 1\right)$$



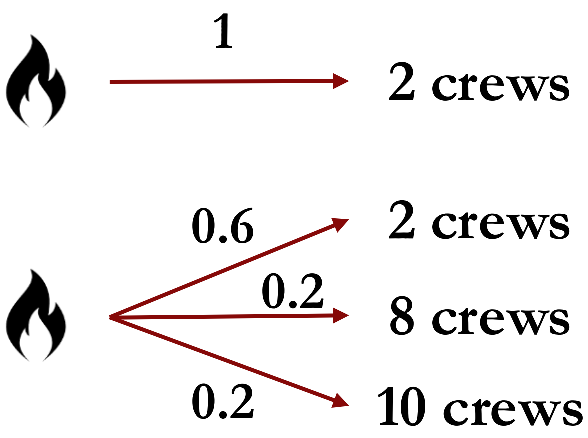

**Theorem: Dual-aware max-variance branching converges finitely to integer solution, whereas fractional br... may**

$B_{qt} \leq 5$

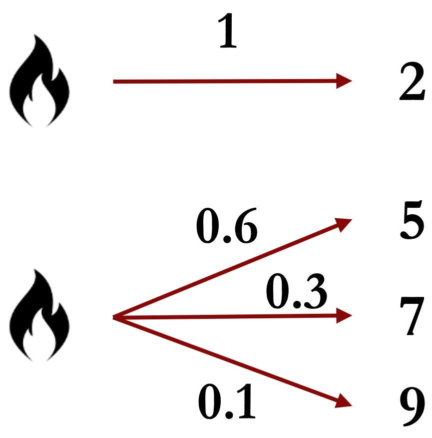
$B_{qt} \geq 6$

# Cutting planes: 10-crew fire suppression plan example

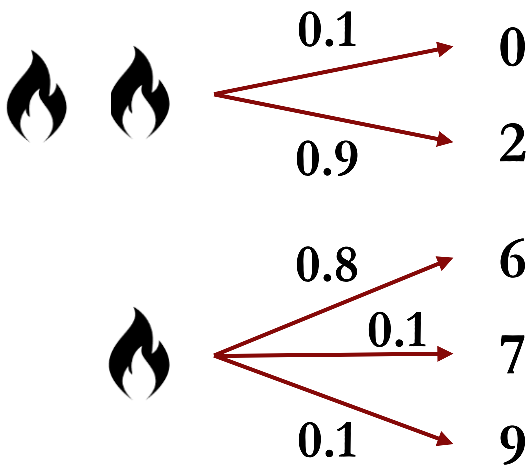
Problem: During master LP relaxation, the solver may pick **fractions of different fire suppression plans.**




$$\sum_{q\in\mathcal{Q}_1}\mathbb{1}[B_{qt}\geq 2]y_q + \sum_{q\in\mathcal{Q}_2}\mathbb{1}[B_{qt}\geq 10]y_q \leq 1$$




$$\sum_{q\in\mathcal{Q}_1}\mathbb{1}[B_{qt}\geq 2]y_q + \sum_{q\in\mathcal{Q}_2}\mathbb{1}[B_{qt}\geq 9]y_q \leq 1$$




$$\frac{1}{2}\sum_{q\in\mathcal{Q}_1}\mathbb{1}[B_{qt}\geq 2]y_q + \frac{1}{2}\sum_{q\in\mathcal{Q}_2}\mathbb{1}[B_{qt}\geq 2]y_q + \frac{1}{2}\sum_{q\in\mathcal{Q}_3}\mathbb{1}[7\leq B_{qt}\leq 8]y_q + \sum_{q\in\mathcal{Q}_3}\mathbb{1}[B_{qt}\geq 9]y_q \leq 1$$

# Cutting planes for LP: robust cuts

- Generalized upper-bound cover cuts (Wolsey)

$$\sum_{g\in\mathcal{G}_u}\sum_{q\in\mathcal{Q}_g}\mathbb{1}[B_{qt}\geq D_{ug}]y_q+\sum_{j\in\mathcal{J}_u}\sum_{p\in\mathcal{P}_j}\mathbb{1}\left[\sum_{g\in\mathcal{G}_u}A_{pgt}=0\right]z_p\leq|\mathcal{G}_u|+|\mathcal{J}_u|-1$$

- Strengthened GUB cover cuts in the column generation context

$$\sum_{g\in\mathcal{G}_u}\sum_{q\in\mathcal{Q}_g}\mathbb{1}[B_{qt}\geq \underline{D}_{ug}]y_q+\sum_{j\in\mathcal{J}_u}\sum_{p\in\mathcal{P}_j}\mathbb{1}\left[\sum_{g\in\mathcal{G}_u}A_{pgt}=0\right]z_p\leq|\mathcal{G}_u|+|\mathcal{J}_u|-1$$

- Augmented GUB cover cuts
  - Augmented GUB closure contained in GUB closure
  - Cut-generating linear program to generate augmented GUB cover cuts

$$\sum_{g\in\mathcal{G}_u}\sum_{q\in\mathcal{Q}_g}\sum_{d=0}^{J}\mathbb{1}[B_{qt}=d]\delta_{ugd}y_q+\sum_{j\in\mathcal{J}_u}\sum_{p\in\mathcal{P}_j}\mathbb{1}\left[\sum_{g\in\mathcal{G}_u}A_{pgt}=0\right]z_p\leq|\mathcal{J}_u|+K_u$$